STATISTICS 3340, Assignment 3, due Sunday, October 6, 11:59 PM.

Name: ______________________________ **Banner #:** B00__________________

1. A regression analysis is to be carried out for the model $y = \beta_0 + \beta_1 X_1 + \epsilon$. There are six observations, with $\boldsymbol{y}' = (3, 2, 1, 1, 2, 0)$ and

$$\mathbf{X} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -2 \\ 1 & -1 & 1 \end{bmatrix}$$

(a) Find $(\boldsymbol{X}'\boldsymbol{X})^{-1}$

(b) Find the (6,6) element of $\boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'$. (We will see later that this is referred to as the "leverage" of the 6'th observation.)

(c) Find the least squares estimator of $\boldsymbol{\beta}$.

You can verify your calculation of $\hat{\boldsymbol{\beta}}$ with the following R code:

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y= c(3, 2, 1, 1, 2, 0)
x1=c(1,1,1,-1,-1,-1)
x2=c(1,-2,1,1,-2,1)
lm(y~x1+x2)
```

2. Suppose that X and Y are both have mean 0 and variance 1, and that the covariance between X and Y is -.5.

   (a) What are the mean and variance of $X + Y$?

   (b) What are the mean and variance of $X - 2Y$?

   (c) What is the covariance between $X + Y$ and $2X - 3Y$?

3. For the data $y = (1, 2, 4)$ and $x = (0, 1, 2)$, in order to to fit the model $y = \beta_0 + \beta_1 x + \epsilon$ using matrix calculations, what is the appropriate matrix $\mathbf{X}$?

4. Suppose X, Y and Z are random variables with means $\mu_X = 1$, $\mu_Y = 2$ and $\mu_Z = 3$, variances $\sigma_X^2 = 9$, $\sigma_Y^2 = 4$ and $\sigma_Z^2 = 1$, and covariances $Cov(X, Y) = 3$, $Cov(Y, Z) = 1$ and $Cov(X, Z) = 1.5$. Let $U = X - Y + 2Z$ and $V = X + 2Y - 2Z$.

   What is $E(U)$?

   What is $Var(U)$?

   What is $Cov(U, V)$?

5. Suppose that Y is a random vector with mean vector $(3, 0, 1)'$, and covariance matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Let $\mathbf{b} = (-3, 0, 1)'$ and

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 2 \\ 3 & 2 & 4 \end{bmatrix}$$

(a) What is the mean of $\mathbf{AY} + \mathbf{b}$?

(b) What is the covariance matrix of $\mathbf{AY} + \mathbf{b}$?

(c) What is the mean of $\mathbf{Y}'\mathbf{AY}$?

6. An extension of the usual multiple regression model is the 'mixed effects' model

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{Z}\boldsymbol{u} + \boldsymbol{\epsilon}$$

The mixed effects model assumes that

- $\boldsymbol{\epsilon}$ is a random vector with mean vector $\mathbf{0}$ and covariance matrix $\sigma^2\boldsymbol{I}$
- $\boldsymbol{u}$ is a random vector with mean vector $\mathbf{0}$ and covariance matrix $\tau^2\boldsymbol{I}$
- $\boldsymbol{u}$ and $\boldsymbol{\epsilon}$ are independent of one another
- $\boldsymbol{X}$, $\boldsymbol{Z}$ are matrices of known constants, and $\boldsymbol{\beta}$ is a vector of constants.

Find $Cov(\boldsymbol{y}, \boldsymbol{u})$, the covariance between y and u.