

STAT 3340 Assignment 4, Fall 2025, due Thursday, November 20,
11:59 PM

1. The regression model $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$ was fit to a data set with $n = 24$ observations, and resulted in $\text{MSE}=16$, with $\hat{\beta}$ and $(X'X)^{-1}$ given by:

```
hatbeta= c(1000, 100, -200); hatbeta
## [1] 1000 100 -200
xpxinverse=matrix(c(15,8,7,8,12,5,7,5,20), byrow=T,ncol=3); xpxinverse
##      [,1] [,2] [,3]
## [1,]    15     8     7
## [2,]     8    12     5
## [3,]     7     5    20
```

- 1a) Construct a 95% confidence interval for β_1 .
- 1b) Construct simultaneous 90% confidence intervals for β_1 and β_2 .
- 1c) Construct a 99% confidence interval for $\beta_0 - 2\beta_1 + \beta_2$.
- 1d) Is the value $\beta = c(950, 90, -230)$ contained in the 95% confidence ellipse for β ? (Hint: Calculate the value of $(\hat{\beta} - \beta)^T (X'X)(\hat{\beta} - \beta)$ and compare it to $pMSE qf(.95, p, n - p)$.) Show your work.

2. In a study of 1006 men in four occupations, a multiple regression was carried out to show how lung function was related to age, smoking and occupation. The four occupations represented in the study were physician, firefighter, farm worker, and chemical worker. The variables in the regression were:

Y air capacity (ml) that the worker can expire in one second
 X_1 age in years
 X_2 number of cigarettes smoked per day
 X_3 1 if chemical worker, 0 otherwise
 X_4 1 if farm worker, 0 otherwise
 X_5 1 if firefighter, 0 otherwise

The model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + e$ was fit to the data with the following result:

$$\hat{Y} = 4500 - 39X_1 - 9X_2 - 350X_3 - 380X_4 - 180X_5$$

- 2a) What is the mean air capacity of a 20 year old farmworker who smokes 10 cigarettes per day?
- 2b) What is the estimated difference in air capacity between two farmers of the same age, one of whom smokes 10 cigarettes per day, and the other of whom does not smoke?

3. Prove that $\mathbf{H}\mathbf{X} = \mathbf{X}$. (Hint: this is one line of algebra.) (2)