STATISTICS 3340, Assignment 6 solutions

(3) 1. A regression of Y on three predictors X_1, X_2, and X_3 was carried out. There were 100 observations. The total sum of squares was 32.5, and the R-squared value was $R^2 = .7$ What is the value of R^2_{adj}?

$SSE = (1 - R^2)SST = .3(32.5) = 9.75$

Then,

$$R^2_{adj} = 1 - \frac{SSE/(n-p)}{SST/(n-1)} = 1 - \frac{9.75/96}{32.5/99} \approx .691$$

or equivalently,

$$R^2_{adj} = 1 - \left(\frac{n-1}{n-p}\right)(1 - R^2) = 1 - \frac{99}{96}(.3)$$

(3) 2. In a regression problem, the deleted residual for case i is $e_{(i)} = 2.00$, and the leverage value for that case is $h_{ii} = .3$. What is the usual (undeleted) residual e_i for that case?

$$e_i = e_{(i)}(1 - h_{ii}) = 2(1 - .3) = 1.4$$

(3) 3. In a different regression problem, the estimated variance when the ith case is deleted was $s^2_{(i)} = 2.25$. If the raw residual for that case is $e_i = 3.75$ and $h_{ii} = .3$, what is the value of the externally studentized residual for case i?

$$t_i = \frac{e_{(i)}/(1 - h_{ii})}{s_{(i)}/\sqrt{1 - h_{ii}}} = \frac{3.75/.7}{\sqrt{2.225}/\sqrt{.7}} \approx 2.988$$

or more directly,

$$t_i = \frac{e_i}{s_{(i)}\sqrt{1 - h_{ii}}}$$

4. Suppose that you wish to test that β_1, β_2 and β_3 are all equal, and that $\beta_0 = 1$, using a null hypothesis of the form $H_0 : \boldsymbol{T\beta} = \boldsymbol{c}$.

(3) (a) What is $\boldsymbol{T}$?

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

T is not unique. There are several equivalent choices.

(2) (b) What is $\boldsymbol{c}$?

For the T indicated, the appropriate value of $\boldsymbol{c}$ is

$$c = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

5. Suppose that Y is a Bernoulli random variable with parameter p, for which the mean of Y is p and the variance of Y is $p(1-p)$. If $h(Y) = arcsin(\sqrt{Y})$,

(2) (a) What is the approximate mean of $h(Y)$?

$$E(h(Y)) \approx h(E(Y)) = arcsin(\sqrt{p})$$

(4) (b) What is the approximate variance of $h(Y)$?

$$h^{'}(Y) = \frac{d}{dY}h(Y) = \frac{1}{2}\frac{Y^{-1/2}}{\sqrt{1-(Y^{1/2})^2}}$$

$$V(h(Y)) \approx \left(\frac{1}{2\sqrt{p}\sqrt{1-p}}\right)^2 p(1-p) = \frac{1}{4}$$

(5) 6. Suppose that x and y have a nonlinear relationship of the form

$$y = \left(\frac{\beta_0 x}{\beta_1 + \beta_2 x}\right)^{\frac{1}{3}}$$

Find transformations of y and/or x such that the relationship between the transformed variables is linear.

$$y^3 = \left(\frac{\beta_0 x}{\beta_1 + \beta_2 x}\right)$$

$$\frac{1}{y^3} = \frac{\beta_1 + \beta_2 x}{\beta_0 x} = \frac{\beta_2}{\beta_0} + \frac{\beta_1}{\beta_0}\frac{1}{x}$$

Let $y' = \frac{1}{y^3}$ and $x' = \frac{1}{x}$. Then the above equation is

$$y' = \frac{\beta_2}{\beta_0} + \frac{\beta_1}{\beta_0}x'$$

and y' is a linear function of x' with intercept $\frac{\beta_2}{\beta_0}$ and slope $\frac{\beta_1}{\beta_0}$.