Matrix approach to Simple Linear Regression

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$
$$i = 1, 2, \ldots, n$$

$$\underset{\sim}{y}' = \langle y_1, y_2, \ldots, y_n \rangle$$

$$\underset{\sim}{\varepsilon}' = \langle \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n \rangle$$

$$\underset{\sim}{\beta}' = \langle \beta_0, \beta_1 \rangle$$

$$\boxed{\begin{aligned} \underset{\sim}{y} &= X\underset{\sim}{\beta} + \underset{\sim}{\varepsilon} \\ \hat{\underset{\sim}{\beta}} &= \\ &(X'X)^{-1} X' \underset{\sim}{y} \end{aligned}}$$

$$X = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & x_3 \\ & \vdots \\ 1 & x_n \end{pmatrix}$$

$$X'X = \begin{pmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{pmatrix}$$

$$X'y = \begin{pmatrix} \Sigma y_i \\ \Sigma x_i y_i \end{pmatrix}$$

$$(X'X)^{-1} = \frac{1}{n \Sigma x_i^2 - (\Sigma x_i)^2} \begin{pmatrix} \Sigma x_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{pmatrix}$$

$$\hat{\underset{\sim}{\beta}} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix} = (X'X)^{-1} X'y$$

$$\hat{\beta}_1 = \frac{n \Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2}$$

$$\hat{\beta}_0 = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n \sum x_i^2 - \left(\sum x_i\right)^2} = \frac{A}{B}$$

$$A = n \sum x_i^2 \bar{y} - \left(\sum x_i\right)^2 \bar{y} + \left(\sum x_i\right)^2 \bar{y}$$

$$- \sum x_i \sum x_i y_i$$

$$= \bar{y} B - \bar{x}\left(n \sum x_i y_i - \sum x_i \sum y_i\right)$$

$$= \bar{y} B - \bar{x} \hat{\beta}_1 B$$

$$\Rightarrow \hat{\beta}_0 = \frac{A}{B} = \bar{y} - \hat{\beta}_1 \bar{x}$$