

TABLE 5.4 Linearizable Functions and Corresponding Linear Form

Figure	Linearizable Function	Transformation	Linear Form
5.4a, b	$y = \beta_0 x^{\beta_1}$	$y' = \log y, x' = \log x$	$y' = \log \beta_0 + \beta_1 x'$
5.4c, d	$y = \beta_0 e^{\beta_1 x}$	$y' = \ln y$	$y' = \ln \beta_0 + \beta_1 x$
5.4e, f	$y = \beta_0 + \beta_1 \log x$	$x' = \log x$	$y' = \beta_0 + \beta_1 x'$
5.4g, h	$y = \frac{x}{\beta_0 x - \beta_1}$	$y' = \frac{1}{y}, x' = \frac{1}{x}$	$y' = \beta_0 - \beta_1 x'$

Other models that can be linearized by reciprocal transformations are

$$\frac{1}{y} = \beta_0 + \beta_1 x + \varepsilon$$

and

$$y = \frac{x}{\beta_0 x - \beta_1 + \varepsilon}$$

This last model is illustrated in Figures 5.4g, h.

When transformations such as those described above are employed, the least-squares estimator has least-squares properties with respect to the transformed data, not the original data. For additional reading on transformations, see Atkinson [1983, 1985], Box, Hunter, and Hunter [1978], Carroll and Ruppert [1985], Dolby [1963], Mosteller and Tukey [1977, Chs. 4–6], Myers [1990], Smith [1972], and Tukey [1957].

Example 5.2 The Windmill Data

A research engineer is investigating the use of a windmill to generate electricity. He has collected data on the DC output from his windmill and the corresponding wind velocity. The data are plotted in Figure 5.5 and listed in Table 5.5.

Inspection of the scatter diagram indicates that the relationship between DC output (y) and wind velocity (x) may be nonlinear. However, we initially fit a straight-line model to the data. The regression model is

$$\hat{y} = 0.1309 + 0.2411x$$

The summary statistics for this model are $R^2 = 0.8745$, $MS_{Res} = 0.0557$, and $F_0 = 160.26$ (the P value is <0.0001). Column A of Table 5.6 shows the fitted values and residuals obtained from this model. In Table 5.6 the observations are arranged in order of increasing wind speed. The residuals show a distinct pattern, that is, they move systematically from negative to positive and back to negative again as wind speed increases.

A plot of the residuals versus \hat{y}_i is shown in Figure 5.6. This residual plot indicates model inadequacy and implies that the linear relationship has not captured all of the information in the wind speed variable. Note that the curvature that was