# Differentiating with respect to a vector

Let $\mathbf{A}$ be a symmetric $k \times k$ matrix of constants, $\mathbf{a}$ be a $k \times 1$ vector of constants, and $\boldsymbol{\beta}$ be a $k \times 1$ vector of variables.

- Let $z = \mathbf{a}^T\boldsymbol{\beta}$. The vector of partial derivatives of $z$ with respect to the elements of $\boldsymbol{\beta}$ is

$$\frac{\partial z}{\partial \boldsymbol{\beta}} = \frac{\partial \mathbf{a}^T\boldsymbol{\beta}}{\partial \boldsymbol{\beta}} = \left(\frac{\partial z}{\partial \beta_1}, \frac{\partial z}{\partial \beta_2}, \ldots, \frac{\partial z}{\partial \beta_k}\right)^T = \mathbf{a}$$

- Let $z = \boldsymbol{\beta}^T\mathbf{A}\boldsymbol{\beta}$. The vector of partial derivatives of $z$ with respect to the elements of $\boldsymbol{\beta}$ is

$$\frac{\partial \boldsymbol{\beta}^T\mathbf{A}\boldsymbol{\beta}}{\partial \boldsymbol{\beta}} = 2\mathbf{A}\boldsymbol{\beta}$$