1. $X'X = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ $X'y = \begin{pmatrix} 16 \\ 12 \\ 12 \end{pmatrix}$

$$(X'X)^{-1} = \begin{pmatrix} 1/8 & 0 & 0 \\ 0 & 1/8 & 0 \\ 0 & 0 & 1/8 \end{pmatrix}$$

$$\hat{\underset{\sim}{\beta}} = (X'X)^{-1}X'y = \begin{pmatrix} 16/8 \\ 12/8 \\ 12/8 \end{pmatrix} = \begin{pmatrix} 2 \\ 3/2 \\ 3/2 \end{pmatrix}$$

2. a) true
   b) true, the variances are σ^2
   c) true
   d) false, they are fixed numbers, not random variables

3. a) $\frac{(12+12)}{2} = 12$

b) $V(X) + V(Y) - 2\,\mathrm{cov}(X, Y)$

$= 9 + 9 - 2(8) = 2$

c) $\mathrm{cov}(X - Y, \frac{X}{2} + \frac{Y}{2})$

$= \frac{1}{2}\mathrm{cov}(X, X) + \frac{1}{2}\mathrm{cov}(X, Y) - \frac{1}{2}\mathrm{cov}(X, Y)$

$- \frac{1}{2}\mathrm{cov}(Y, Y)$

$= \frac{1}{2}(8) + \frac{1}{2}(8) - \frac{1}{2}(8) - \frac{1}{2}(8) = 0$

4 a) $\tilde{\beta}_0 \pm t_{.05,18}\sqrt{MSE}\sqrt{C_{11}}$ (*)

$MSE = \frac{SSE}{17-2} = \frac{72}{18} = 4$

$C_{11} = 50/900$

$t_{.05,18} \equiv qt(.95, 18) = 1.734$

$\hat{\beta}_0 = 1.5$

just plug in to (*)

4 b)

$$\underset{\sim}{x}_0 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \qquad \underset{\sim}{\hat{\beta}} = \begin{pmatrix} 1.5 \\ 2 \end{pmatrix}$$

$$\hat{y}_0 = \underset{\sim}{x}_0' \underset{\sim}{\hat{\beta}} = 1.5 - 4 = -2.5$$

$$\hat{y}_0 \pm t_{.05,18} \sqrt{MSE} \sqrt{\underset{\sim}{x}_0' (X'X)^{-1} \underset{\sim}{x}_0} \qquad (**)$$

$MSE = 4$

$t_{.95,18} = 1.734$

$$\underset{\sim}{x}_0' (X'X)^{-1} \underset{\sim}{x}_0 =$$

$$\frac{1}{900} \begin{pmatrix} 1 & -2 \end{pmatrix} \begin{pmatrix} 50 & -10 \\ -10 & 20 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$= \frac{1}{900} \begin{pmatrix} 70 & -50 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \frac{170}{900}$$

just plug into (**)

5a) $X_1 = 20$
$X_2 = 10$
$X_3 = 0$ $\quad X_4 = 1 \quad X_5 = 0$

$4500 - 39(20) - 9(10) - 380(1)$

b) $\pm 9(10) \qquad \pm 90$

c) $\hat{B}_5 \pm t_{.025,1000}\sqrt{MSE}\sqrt{C_{55}}$

$\hat{B}_5 = -180$

$MSE = 9$

$C_{55} = 4$

d) $X_1 = 30 \quad X_2 = 0 \quad X_3 = 1 \quad X_4 = 0 \quad X_5 = 0$

$\underset{\sim}{x}' = (1 \ 30 \ 0 \ 1 \ 0 \ 0)$

$\underset{\sim}{x}'\hat{\underset{\sim}{B}} \pm t_{.025,1000}\sqrt{MSE}\sqrt{\underset{\sim}{x}'(X'X)^{-1}\underset{\sim}{x}}$

$\underset{\sim}{x}'\hat{\underset{\sim}{B}} = 4500 - 39(30) - 350(1)$

$\underset{\sim}{x}'(X'X)^{-1}\underset{\sim}{x}$ not easily done by hand

5e. use $\underset{\sim}{x}' = (1\ 0\ 0\ 0\ 0\ 1)$

$\underset{\sim}{x}'\hat{\underset{\sim}{\beta}} = 4500(1) - 180(1) = 4320$

$\underset{\sim}{x}'(X'X)^{-1} = (16\ 11\ 11\ 7\ 12\ 11)$

$\underset{\sim}{x}'(X'X)^{-1}\underset{\sim}{x} = 16 + 11 = 27$

$\underset{\sim}{x}'\hat{\underset{\sim}{\beta}} \pm t_{.025,1000}\sqrt{9}\sqrt{27}$

6. a) $HX = X(X'X)^{-1}X'X = XI = X$

b) ~~[illegible]~~ $X\begin{pmatrix}1\\0\\0\\0\\0\\\vdots\\0\end{pmatrix} = \underset{\sim}{1}$

from a)

$$HX\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix} = X\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}$$

$\Rightarrow \quad H\underset{\sim}{1} = \underset{\sim}{1}$

c) $H(I - H) = HI - H^2 = H - H^2 = H - H = 0$

d) $\text{cov}(Hy, (I-H)y) = H\,\text{cov}(y)(I-H)$
$= H\sigma^2 I(I-H) = \sigma^2 H(I-H) = 0$

7 a) $E(y) = E(X\underline{\beta}) + E(Z\underline{u}) + E(\underline{\varepsilon})$

$= X\underline{\beta} + Z E(\underline{u}) + E(\underline{\varepsilon})$

$= X\underline{\beta} + Z\underline{0} + \underline{0} = X\underline{\beta}$

b) $\text{cov}(\underline{y}) = \text{cov}(X\underline{\beta} + Z\underline{u} + \underline{\varepsilon})$

$= \text{cov}(Z\underline{u} + \underline{\varepsilon})$ because $X\underline{\beta}$ fixed

$= \text{cov}(Z\underline{u} + \underline{\varepsilon}, Z\underline{u} + \underline{\varepsilon})$

$= \text{cov}(Z\underline{u}, Z\underline{u}) + \text{cov}(Z\underline{u}, \underline{\varepsilon})$

$+ \text{cov}(\underline{\varepsilon}, Z\underline{u}) + \text{cov}(\underline{\varepsilon}, \underline{\varepsilon})$

$= Z\,\text{cov}(\underline{u}, \underline{u}) Z' + Z\,\text{cov}(\underline{u}, \underline{\varepsilon})$

$+ Z\,\text{cov}(\underline{\varepsilon}, \underline{u}) + \text{cov}(\underline{\varepsilon}, \underline{\varepsilon})$

$= Z \tau^2 I Z' + 0 + 0 + \sigma^2 I$

$= \tau^2 Z Z' + \sigma^2 I$

$\text{cov}(\underline{u}, \underline{\varepsilon}) = 0$ by independence

8.

i	x_i	y_i	$\hat{y}_i$	$r_i = y_i - \hat{y}_i$
1	1	4	5	-1
2	2	6	9	-3
3	3		13	4

$r_3 = 4$ so that residuals sum to 0

$\Rightarrow y_3 = 17$